Simplify each rational expression. State any restrictions on the variable(s).

1. \( \frac{x^2-6x-16}{x^2+5x+6} \)  
2. \( \frac{-9x^4y}{27x^3y} \)  
3. \( \frac{-6x^3-3x^2}{x^3-6x^2+8x} \)

Perform the given operation. Simplify as much as possible. State any restrictions on the variable(s).

4. \( \frac{x^2-2x-24}{x^2+7x+12} \cdot \frac{x^2-1}{x-6} \)  
5. \( \frac{4x^2-2x}{x^2+5x+4} \div \frac{2x}{x^2+2x+1} \)  
6. \( \frac{3x}{x^2-4} + \frac{6}{x+2} \)

7. \( \frac{1}{x^2-1} - \frac{2}{x^2+3x} \)  
8. \( \frac{3x}{x-3} + \frac{x^2+5x}{x^2-9} \)  
9. \( \frac{x^2-x-2}{x^2-5x+2} \div \frac{x^2-x-12}{2x^2+5x-3} \)

10. \( \frac{-3}{x+y} \)  
11. \( \frac{3}{1-(2y)^{-1}} \)

Solve the given rational equation. Identify any extraneous solutions.

12. \( \frac{1}{x} = \frac{5}{x-4} \)  
13. \( \frac{2}{x-3} - \frac{4}{x+3} = \frac{8}{x^2-9} \)  
14. \( \frac{4}{2x-3} = \frac{x}{5} \)

15. \( \frac{10}{2y+8} \cdot \frac{7y+8}{y^2-16} = \frac{-8}{2y-8} \)

16. Write a rational equation that represents the following scenario. Then solve.
   One pump can fill a water cistern twice as fast as a second pump. Working together, the two pumps can fill the cistern in 5 hours. Find how long it takes each pump to fill the cistern when working alone.

17. Write a rational equation that represents the following scenario. Then solve.
   A passenger train travels 392 miles in the same time it takes a freight train to travel 322 miles. If the passenger train travels 20 mph faster than the freight train, find the speed of each train.
18. Write an equation of a **reciprocal function** that has a vertical shift, horizontal shift, and a stretch. State your parent function, and the transformations you are applying to it.

19. Write an equation of a **rational function** that has a vertical asymptote of \( x = 3 \), a hole at \( x = -1 \), and a horizontal asymptote of \( y = 2 \).

20. \( f(x) = \frac{-2}{x+1} - 3 \)
   - Domain: ___________________________
   - Range: ___________________________
   - Vertical Asymptote(s): __________
   - Horizontal Asymptote: __________
   - Holes(s): __________
   - \( x \)-intercept(s): __________
   - \( y \)-intercept: __________

21. \( g(x) = \frac{x^2 + x - 6}{x^2 - x - 2} \)
   - Domain: ___________________________
   - Range: ___________________________
   - Vertical Asymptote(s): __________
   - Horizontal Asymptote: __________
   - Holes(s): __________
   - \( x \)-intercept(s): __________
   - \( y \)-intercept: __________