Part 3: Matrices and Solving Systems

Determine the order of the matrix.

1. \[ \begin{bmatrix} 5 & -3 & 8 & 7 \end{bmatrix} \]
   \( 1 \times 4 \) \text{ matrix}

2. \[ \begin{bmatrix} -7 & 6 & 4 \\ 0 & -5 & 1 \end{bmatrix} \]
   \( 2 \times 3 \)

3. \[ \begin{bmatrix} 2 \\ 3 \end{bmatrix} \]
   \( 2 \times 1 \) \text{ matrix}

4. \[ \begin{bmatrix} -3 & 7 & 15 & 0 \\ 0 & -1 & 3 & 7 \\ 1 & 1 & 6 & -8 \end{bmatrix} \]
   \( 3 \times 4 \)

Write the augmented matrix for the system of linear equations.

5. \[ \begin{align*}
7x + 4y &= 22 \\
5x - 9y &= 15
\end{align*} \]
   \[ \begin{bmatrix} 7 & 4 & 22 \\ 5 & -9 & 15 \end{bmatrix} \]

6. \[ \begin{align*}
9x + 2y - 3z &= 20 \\
-25y + 11z &= -5
\end{align*} \]
   \[ \begin{bmatrix} 9 & 2 & -3 & 20 \\ 0 & -25 & 11 & -5 \end{bmatrix} \]

Write a system of equations represented by the augmented matrix.

7. \[ \begin{bmatrix} 4 & -5 & -1 & 18 \\ -11 & 0 & 6 & 25 \\ 3 & 8 & 2 & -29 \end{bmatrix} \]
   \[ \begin{align*}
4x - 5y - z &= 18 \\
-11x + 6z &= 25 \\
3x + 8y + 2z &= -29
\end{align*} \]

With a calculator: Write the matrix in row-echelon form.

8. \[ \begin{bmatrix} 1 & 1 & 0 & 5 \\ -2 & -1 & 2 & -10 \\ 3 & 6 & 7 & 14 \end{bmatrix} \]
   \[ \begin{bmatrix} 1 & 2 & 3 & \frac{14}{3} \\ 0 & 1 & \frac{20}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & -1 \end{bmatrix} \]
   This is different then ref

With a calculator: Use matrices to solve the system of equations.

9. \[ \begin{align*}
3x - 2y + z &= 15 \\
-x + y + 2z &= -10 \\
x - y - 4z &= 14
\end{align*} \]
   \( (5, -1, -2) \)
10. \[
\begin{align*}
 x + 2y + 2z + 4w &= 11 \\
 3x + 6y + 5z + 12w &= 30 \\
 x + 3y - 3z + 2w &= -5 \\
 6x - y - z + w &= -9
\end{align*}
\]
\[\begin{pmatrix}
-1 \\
1 \\
3 \\
1
\end{pmatrix}
\]
\[\begin{pmatrix}
w
\end{pmatrix}
\]

11. \[
\begin{align*}
 5x - 3y + 2z &= 3 \\
 2x + 4y - z &= 7 \\
 x - 11y + 4z &= 3
\end{align*}
\]
\[
\begin{bmatrix}
1 & 0 & \frac{5}{2w} & 0 \\
0 & 1 & -\frac{3}{2w} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\rightarrow
\text{No Solution}
\]

12. \[
\begin{align*}
 -x + y &= 4 \\
 2x - 4y &= -34
\end{align*}
\]
\[\begin{pmatrix}
9 \\
13
\end{pmatrix}
\]

13. \[
\begin{align*}
 x + 2y - 7z &= -4 \\
 2x + y + z &= 13 \\
 3x + 9y - 36z &= -33
\end{align*}
\]
\[
\begin{bmatrix}
1 & 0 & 3 & 10 \\
0 & 1 & -5 & -7 \\
0 & 0 & 0 & 0
\end{bmatrix}
\rightarrow
\infty
\rightarrow
\text{Solutions}
\]

\[
\begin{align*}
2x - y + 3z &= 24 \\
2y - z &= 14 \\
7x - 5y &= 6
\end{align*}
\]
\[\begin{pmatrix}
8 \\
10 \\
6
\end{pmatrix}
\]

*Must show Gaussian Elimination Steps when testing/quizzing*
For the following application problems, set up a system of equations that represents the scenario. Then, solve by using matrices. These are calculator problems.

15. A small shoe corporation borrowed $1,500,000 to expand its line of shoes. Some of the money was borrowed at 7%, some at 8%, and some at 10%. Use a system of equation to determine how much was borrowed at each rate if the annual interest was $130,500 and the amount borrowed at 10% was four times the amount borrowed at 7%.

\[
\begin{align*}
& x + y + z = 1,500,000 \\
& 0.07x + 0.08y + 0.1z = 130,500 \\
& 4x - z = 0
\end{align*}
\]

$150,000 \text{ @ } 7\%$

$750,000 \text{ @ } 8\%$

$600,000 \text{ @ } 10\%$

16. In the 2004 Women's NCAA Final Four Championship game, UCONN defeated Tennessee by a score of 70 to 61. UCONN won by scoring a combination of two-point baskets, three-point baskets, and one-point free throws. The number of two-point baskets was two more than the number of free throws. The number of free throws was one more than two times the number of three-point baskets. What combination of scoring accounted for UCONN's 70 points?

Two points 3 pts FT
\[
\begin{align*}
& 2x + 3y + z = 70 \\
& x - z = 2 \\
& -2y + z = 1
\end{align*}
\]

17 two-point shots
7 3-point shots
15 free throws

17. In the 2004 presidential election, approximately 118,304,000 voters divided their votes among three presidential candidates. George W Bush received 3,320,000 more votes than John Kerry. Ralph Nader received 0.3% of the votes. Write and solve a system of equations to find the total number of votes cast for each candidate. (calculator)

\[
\begin{align*}
& x + y + z = 118,304,000 \\
& x - y = 3,320,000 \\
& z = 354,912
\end{align*}
\]

Nader - 354,912 votes
Kerry - 57,314,500 votes
Bush - 60,634,500 votes