7.4 Similarity within Right Triangles

Learning Targets: The students will learn
• about the geometric mean
• about the relationships formed by the altitude of a right triangle
• to apply those relationships to problems

A. Geometric Mean (an Algebra idea)

What is the geometric mean of 3 and 12?

\[
\frac{x}{3} = \frac{12}{x} \quad \text{or} \quad \frac{3}{x} = \frac{x}{12}
\]

\[x^2 = 3 \cdot 12 \]

\[x = 6\]

What is the geometric mean of 4 and 18?

\[
\frac{x}{4} = \frac{18}{x} \quad \text{or} \quad \frac{4}{x} = \frac{x}{18}
\]

\[x^2 = 4 \cdot 18 \]

\[x = \sqrt{72} \]

\[x = 6\sqrt{2}\]

B. Altitude and similarity

Theorem 7-3
The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.

C. Corollaries to Theorem 7-3
(using geometric means)

Corollary 1
The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

\[
\frac{a - \text{seg}_1}{x} = \frac{\text{seg}_2}{b}
\]

Use "T" trick
**Corollary 2**
The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the adjacent hypotenuse segment and the length of the hypotenuse. (HUH?????)

\[
\frac{a}{x} = \frac{x}{b}
\]

Problems. Find \(x\) and \(y\).

1. \[x = 4, \quad y = 12\]
   
   \[
   \frac{x}{y} = \frac{\sqrt{x^2 + 12^2}}{y} = \frac{\sqrt{4^2 + 12^2}}{y} = \frac{\sqrt{16 + 144}}{y} = \frac{\sqrt{160}}{y} = \frac{4\sqrt{10}}{y}
   \]
   
   \[
   y = \frac{4\sqrt{10}}{4} = \sqrt{10}
   \]

2. \[x = 6, \quad y = 8\]
   
   \[
   \frac{x}{y} = \frac{\sqrt{x^2 + 8^2}}{y} = \frac{\sqrt{6^2 + 8^2}}{y} = \frac{\sqrt{36 + 64}}{y} = \frac{\sqrt{100}}{y} = \frac{10}{y}
   \]
   
   \[
   y = \frac{10}{10} = 1
   \]

3. \[\frac{10}{8} = \frac{2x + 1}{10}\]
   
   \[
   100 = 8(2x + 1) = 16x + 8
   \]
   
   \[
   92 = 16x
   \]
   
   \[
   x = 5.75
   \]

**Homework**

Pg. 394 #1-3, 9-21 (on 21 just set-up and cross-multiply),

23, 27-30, 34-38