Systems and Matrices Test Review

Vocabulary Review

- Consistent System of equations: More than 1 solution but NOT infinite
- Consistent Dependent System of equations: Infinite solutions
- Consistent Independent System of equations: only 1 solution
- Inconsistent System of equations: No Solution
- Row Echelon Form: 
- Gaussian Elimination: 
- Matrix: 
- Order of a Matrix: 
- Entry of a Matrix: 
- Square Matrix: 
- Row Matrix: 
- Column Matrix: 
- Equality of Matrices: Must be same order
- Scalar Multiplication: 
- Proving two Matrices are Inverses: Show AB and BA equal the Identity
- Determinate of a Matrix: \( ad - bc \)
1. Solve the system of equations. (no calculator)

\[
\begin{align*}
4x - 3y - 2z &= 21 \\
6y - 5z &= -8 \\
\quad z &= -2
\end{align*}
\]

\[ (2, -3, -2) \]

2. Using Gaussian Elimination, solve the system of equations. (no calculator)

\[
\begin{align*}
x + y + z &= 6 \\
2x - y + z &= 3 \\
3x - z &= 0
\end{align*}
\]

Must use statements like: \( 2R_1 - R_2 \rightarrow R_2 \)

to organize work.

3. Using Gaussian Elimination, solve the system of equations. (no calculator)

\[
\begin{align*}
3x + y - z &= 4 \\
x + 2y + 2z &= 5 \\
4x + y - z &= 3
\end{align*}
\]

Example step 1: \( R_1 - 3R_2 \rightarrow R_2 \)
4. Solve the system of equations. *(calculator)*

\[
\begin{align*}
2x + y - z &= 7 \\
x - 2y + 2z &= -9 \\
3x - y + z &= 5
\end{align*}
\]

ref \[
\begin{bmatrix}
2 & 1 & -1 & 7 \\
1 & -2 & 2 & -9 \\
3 & -1 & 1 & 5
\end{bmatrix}
\]

No Solution

5. Solve the system of equations. *(calculator)*

\[
\begin{align*}
-6x - y + 4z &= -19 \\
-2x + 5y - 4z &= 15 \\
-6x + 2y + z &= -7
\end{align*}
\]

\[
\text{Infinite Solutions}
\]

6. Write a system of equations that represents the following scenario. Then solve. *(calculator)*

A small corporation borrowed $775,000 to expand its clothing line. Some of the money was borrowed at 8%, some at 9%, and some at 10%. How much was borrowed at each rate if the annual interest owed was $67,500 and the amount borrowed at 8% was four times the amount borrowed at 10%?

$300,000 \text{ borrowed @} 8\%$

$400,000 \text{ borrowed @} 9\%$

$150,000 \text{ borrowed @} 10\%$

7. Write a system of equations that represents the following scenario. Then solve. *(calculator)*

A florist is creating 10 centerpieces for a wedding. The florist can use roses that cost $2.50 each, lilies that cost $4 each, and irises that cost $2 each to make the bouquets. The customer has a budget of $300 and wants each bouquet to contain 12 flowers, with twice as many roses used as the other two types of flowers combined. How many of each type of flower should be in each centerpiece?

80 Roses

10 Lilies

30 Irises

How many total flowers?
8. Use the matrix \( A = \begin{bmatrix} 5 & 1 & -11 & 4 \\ -2 & 7 & 3 & 12 \\ 0 & -9 & 6 & 8 \end{bmatrix} \) to answer the following questions. (no calculator)

a) What is the order of matrix \( A \)?

\[ \frac{3 \times 4}{11 \quad -9 \quad 12} \]

b) What is the value of the entry: \( a_{13} \)

9. Using the equation \[ \begin{bmatrix} x+5 & -1 & 20 \\ 7 & -8z & 3 \\ 1 & 5-y & 2 \end{bmatrix} = \begin{bmatrix} 2x+3 & -1 & 20 \\ 7 & -16 & 3 \\ 1 & 11+z & 2 \end{bmatrix} \], find the values of the variables.

(no calculator)

\[ x = 2 \quad , \quad y = -8 \quad , \quad z = 2 \]

10. Suppose \( A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & 4 \end{bmatrix} \) and \( B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix} \). Solve for \( X \) in the equation given. (no calculator)

\[ \frac{2X = 2A - B}{2} \]

so \( X = \frac{2A - B}{2} \) or \( A - \frac{1}{2}B \)

\[ \begin{bmatrix} -2 & -\frac{5}{2} \\ 0 & 0 \\ 5 & \frac{9}{2} \end{bmatrix} \]

11. Evaluate the expression if possible. (no calculator)

\[ -3 \left( \begin{bmatrix} 0 & 3 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ 8 & 1 \end{bmatrix} \right) - 2 \left( \begin{bmatrix} 4 & -4 \\ 7 & -9 \end{bmatrix} \right) = \begin{bmatrix} 10 & 8 \\ -59 & 9 \end{bmatrix} \]

Show steps along the way for partial credit
12. Evaluate the expression if possible. \((no\ calculator)\)

a) \[
\begin{bmatrix}
1 & 5 & 6 \\
2 & -4 & 0 \\
2 & 3 & 3
\end{bmatrix}
\times
\begin{bmatrix}
6 & 4 \\
-2 & 0 \\
8 & 0
\end{bmatrix}
\]

b) \[
\begin{bmatrix}
-6 & 3 \\
8 & 1
\end{bmatrix}
\times
\begin{bmatrix}
5 & -6 \\
-2 & 0 \\
8 & 1
\end{bmatrix}
\]

13. Find the inverse of the following matrix if possible. \((no\ calculator)\) → Show Work

a) \(B = \begin{bmatrix} -7 & 2 \\ -8 & 2 \end{bmatrix}\)

\(B^{-1} = \begin{bmatrix} 1 & -1 \\ 4 & -\frac{7}{2} \end{bmatrix}\)

b) \(C = \begin{bmatrix} -4 & 2 \\ -8 & 4 \end{bmatrix}\)

\(C^{-1} = \text{undefined}\)

\(\text{determinant} = 0\)

No Inverse; \(C\) is a singular matrix

14. Prove that \(B\) is an inverse of \(A\). \((no\ calculator)\)

\(A = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}\); \(B = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}\)

Show Work for

\(AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\) and \(BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\)
15. At a convenience store, the numbers of gallons of 87-octane, 89-octane, and 93-octane sold over the weekend are represented by matrix $A$.

\[
A = \begin{bmatrix}
580 & 840 & 320 \\
560 & 420 & 160 \\
860 & 1020 & 540
\end{bmatrix}
\]

The selling prices per gallon and the profits per gallon for the three grades of gasoline sold by the convenience store are represented by $B$.

\[
B = \begin{bmatrix}
1.95 & 0.32 & \text{Types of Octane} \\
2.05 & 0.36 & \text{87} \\
2.15 & 0.40 & \text{89}
\end{bmatrix}
\]

Compute $AB$ and then interpret the results. (calculator)

\[
AB = \begin{bmatrix}
3541 & 6116 \\
2297 & 394.4 \\
4929 & 858.4
\end{bmatrix}
\]

Column 1 is selling prices per day.
Column 2 is profits per day.

Find the convenience store’s profit from gasoline sales for the weekend. (calculator)

Add the profit column: \(6116 + 394.4 + 858.4\)

\[\$1,868.80\]
16. The numbers of calories burned by individuals of different body weights performing different types of aerobic exercises for a 20-minute period are shown in matrix A.

\[
A = \begin{bmatrix}
109 & 136 \\
127 & 159 \\
64 & 79
\end{bmatrix}
\]

Since each entry is based on a 20-minute period; a 40 min workout is two periods.

A 125-pound person and a 150-pound person biked for 40 minutes, jogged for 10 minutes, and walked for 60 minutes. Create a matrix, named B, that represents the time spent exercising by each person.

\[
B = \begin{bmatrix}
2 & \frac{1}{2} & 3 \\
\end{bmatrix}
\]

Compute \(BA\) and interpret the results. (calculator)

Matrix B must have 3 columns in order to mult. by A.

\[
\begin{bmatrix}
473.5 & 588.5
\end{bmatrix}
\]

Matrix B must have 3 columns in order to mult. by A. 

Number calories burned for each person based on time they spent working out.

Written Response Questions

You should be able to answer/analyze/explain questions similar to the following.

a) Will the following statement always be true? \(AB = BA\) → only if A and B are inverses.

b) Given a specific scenario, explain how a matrix/matrices outcome will appear.

c) If \(A = B^{-1}\), then will \(AB = BA\)?

Yes \(AB = BA\) because a matrix times its inverse will always be the identity regardless of the order of multiplication.